ψ' to ψ Ratio in Diffractive Photoproduction

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We evaluate the ψ' to J/ψ ratio in diffractive photoproduction in a light-cone framework, using charmonium wave functions extracted from non-relativistic potential models. Contrary to current belief, we find that the best estimate for the ratio is a factor 2 to 5 below the data. The measured ratio constrains the distribution of the $c\bar{c}$ component of the charmonium light-cone wave function and indicates that it is more compact than in potential models. We predict that the inelastic photoproduction ratio will be bigger than the elastic one, and will equal that measured in hadroproduction.

I. INTRODUCTION

Quarkonium production is a hard process in which the heavy quarks are produced with limited relative momentum. This kinematic restriction implies that the standard QCD factorization theorem does not apply, *ie.*, there is no guarantee that the cross section can be expressed as a product of universal parton distributions and a partonic subprocess. Quarkonium production is thus sensitive to the environment and yields new information about the dynamics of hard processes. The abundant data imposes severe restrictions on theoretical models (for a discussion see Ref. [1]). In particular, the recent CDF data [2] on charmonium polarization at large $p_\perp$ in $p\bar{p}$ collisions disagrees with the color octet model prediction [3].

The ratio of cross sections for radially excited states is sensitive to the time-scale of the production process. If all relevant interactions occur while the quark pair is compact, $r_\perp \sim 1/m_Q$, the direct production cross section is proportional to $|\Phi(0)|^2$, the square of the quarkonium wave function at the origin. Conversely, late interactions depend on the shape of the wave function out to its Bohr radius, $r_\perp \sim \mathcal{O}(1/(\alpha_s m_Q))$. The $\sigma(\psi')/\sigma(J/\psi)$ ratio is thus a very interesting quantity which may give clues on the correct production mechanism.

In forward charmonium hadroproduction the ratio of ψ' to J/ψ cross sections is found to be roughly independent of the kinematics and of the size of the nuclear target [4,5]. Its value is moreover consistent with the expectation based on the proportionality to $|\Phi(0)|^2$ [6],

$$R^{hN} \equiv \frac{\sigma^{hN}(\psi')}{\sigma_{dir}^{hN}(J/\psi)} = \frac{\Gamma(\psi' \to e^+ e^-)}{\Gamma(J/\psi \to e^+ e^-)} \frac{M_{J/\psi}^3}{M_{\psi'}^3} \simeq 0.24 \,, \tag{1}$$

suggesting that late interactions are unimportant for quarkonia produced in hadron fragmentation regions. In nuclear fragmentation regions the ratio is measured to be smaller than in Eq. (1). This is explained by the larger break-up cross section of the ψ' in late interactions with the nuclear comovers.

Preliminary CDF results [2] show that $52 \pm 12\%$ of the $\Upsilon(1S)$ are directly produced. Given that the $\Upsilon(3S)$ is produced only directly one deduces from the published total cross sections [7] that $\sigma(3S)/\sigma_{dir}(1S) = 0.4 \pm 0.1$, consistent with the expectation 0.3 ± 0.05 based on Eq. (1) for bottomonium.

Diffractive charmonium electroproduction, $\gamma^{(*)}p \to \psi(nS)p$, is believed to occur via two-gluon exchange [8]. At large Q^2, the size of the quark pair in the virtual photon wave function is $\sim 1/Q \ll 1/(\alpha_s m_c)$, and the cross section is predicted to be proportional to $|\Phi_n(0)|^2$. The ψ' to J/ψ ratio at large Q is indeed measured [9] to be consistent with $|\Phi_{\psi'}(0)/\Phi_{J/\psi}(0)|^2 \sim 0.5 - 0.6$ [10]. On the other hand, for photoproduction [11] the measured value

$$R_{el}^{\gamma N} \equiv \frac{\sigma(\gamma p \rightarrow \psi' p)}{\sigma(\gamma p \rightarrow J/\psi p)} = 0.15 \pm 0.034 \quad (Q^2 = 0) \quad (2)$$

is about a factor 3 below the value found at large Q^2.

It was pointed out [12] that due to the moderate value of the charm quark mass and a factor $r_\perp^2$ from the coupling of the two gluons, the photoproduction amplitude in fact probes the charmonium wave function at a transverse size $r_\perp$ which is comparable to the Bohr radius. This reduces the ψ' contribution due to the node in its wave function. The range of transverse size which is probed decreases when the virtuality Q^2 of the photon increases. It is thus possible to measure the shape of the charmonium wave function using electroproduction data.

In Ref. [12] an estimate of the node effect gave a value .15 for the ratio (2).

This value was obtained with harmonic oscillator wave functions for the bound states and with the weighted photon wave function parametrized as a sum of two Gaussian functions [13]. We find that the result is very sensitive to the parametrization and is thus uncertain to at least 50%.

We report here a more quantitative calculation of the photoproduction ratio, in a light-cone framework using charmonium wave functions obtained from potential models. Surprisingly, the calculated ratio (2) turns out to be a factor 2 to 5 below the data. We discuss the implications of this for the charmonium wave function.

II. EVALUATION OF THE RATIO

The measured $\psi'/J/\psi$ photoproduction ratio is consistent with being independent of the photon energy [11,14]. At the H1 energy the incoming photon fluctuates into the $c\bar{c}$ pair long before the target. We therefore work in the high energy regime where the transverse size $r_\perp$ of the (S-wave) $c\bar{c}$ pair is frozen during its interactions in the target and distributed according to the photon wave function

$$\Phi_\gamma(x, \boldsymbol{r}_\perp) \propto \sqrt{x(1-x)}\, K_0(m_c r_\perp) , \quad (3)$$

where x is the light-cone momentum fraction carried by the c quark.

Each exchanged gluon couples to the $c\bar{c}$ pair with a strength proportional to the color dipole length $r_\perp$ (in the usual approximation where the gluon wavelengths are long compared to $r_\perp$). The forward scattering amplitude is then given [15] by an overlap with the charmonium wave function Φ_ψ:

$$\mathcal{M}_\psi \propto \int \frac{dx}{\sqrt{x(1-x)}} d^2 \boldsymbol{r}_\perp \Phi_\gamma(x, \boldsymbol{r}_\perp) r_\perp^2 \Phi_\psi(x, \boldsymbol{r}_\perp)^* . \quad (4)$$

We consider two models for Φ_ψ, obtained from the non-relativistic Buchmüller-Tye (BT) and Cornell potentials, respectively [10]. In the non-relativistic limit

there is a simple relation between the light-cone amplitude $\Phi_\psi(x, \boldsymbol{r}_\perp)$ appearing in Eq. (4) and the equal-time wave function $\Phi_\psi^{NR}(\boldsymbol{r})$ given by the Schrödinger equation (see, eg., Ref. [16]). In momentum space,

$$\Phi_\psi(x, \boldsymbol{p}_\perp) = \frac{2(p^2 + m_c^2)^{3/4}}{(p_\perp^2 + m_c^2)^{1/2}} \Phi_\psi^{NR}(\boldsymbol{p}) , \quad (5)$$

where

$$x = \frac{1}{2} + \frac{p^z}{2\sqrt{p^2 + m_c^2}} \quad (6)$$

and the relative factor is fixed by the normalization conditions.

Using

$$F(\boldsymbol{p}_\perp) \equiv -\int \frac{d^2 \boldsymbol{r}_\perp}{8\pi} e^{-i\boldsymbol{p}_\perp \cdot \boldsymbol{r}_\perp} r_\perp^2 K_0(m_c r_\perp)$$

$$= \frac{p_\perp^2 - m_c^2}{(p_\perp^2 + m_c^2)^3} , \quad (7)$$

Eq. (4) reads in momentum space

$$\mathcal{M}_\psi \propto \int d^3\boldsymbol{p} \frac{(p_\perp^2 + m_c^2)^{1/2}}{(p^2 + m_c^2)^{3/4}} F(\boldsymbol{p}_\perp) \Phi_\psi^{NR}(\boldsymbol{p})^* . \quad (8)$$

We use the Mathematica program of Lucha and Schöberl [17] to solve the Schrödinger equation for the BT and Cornell potentials. Our results for the cross section ratio (2),

$$R_{el}^{\gamma N} = \frac{|\mathcal{M}_{\psi'}|^2}{|\mathcal{M}_{J/\psi}|^2} . \quad (9)$$

are shown in the first line of Table I. The squared ratio of wave functions at the origin shown on the second line is the result predicted for highly virtual photons.

$R_{el}^{\gamma N} = \sigma_{\psi'}/\sigma_{J/\psi}$	BT	Cornell
Eq. (8)	0.033	0.070
$\|\Phi_{\psi'}^{NR}(0)/\Phi_{J/\psi}^{NR}(0)\|^2$	0.65	0.64
$r_\perp^2 \rightarrow r_\perp$	0.19	0.23

TABLE I. The ψ' to J/ψ ratio calculated with the Buchmüller-Tye (BT) and Cornell wave functions [10] for elastic photoproduction (first line), large Q^2 electroproduction (second line), and inelastic photoproduction (last line).

III. DISCUSSION

Eq. (4) gives a cross section ratio which is an order of magnitude smaller than the ratio of wave functions at
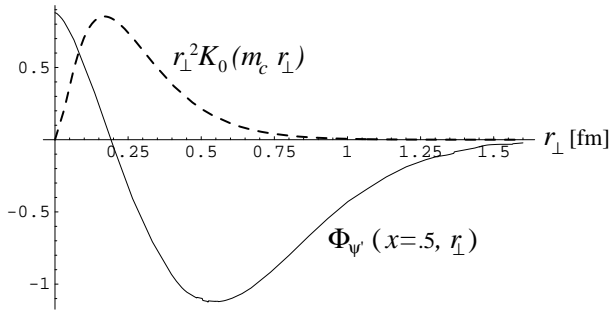
FIG. 1. The light-cone ψ' wave function $\Phi_{\psi'}(x = .5, r_\perp)$ (solid line) and the weighted photon wave function $r_\perp^2 K_0$ (dashed line) appearing in the integrand of Eq. (4), plotted versus $r_\perp$. The normalizations are arbitrary.

the origin. This means that the photon wave function, weighted by $r_\perp^2$, probes the charmonium wave functions at relatively large separations. As seen from Fig. 1, $r_\perp^2 \Phi_\gamma$ in fact gives similar weights to the ψ' wave function in the regions below and above the node, leading to a near cancellation in the ψ' integral. This is the reason for the small value of $R_{el}^{\gamma N}$ and for its sensitivity to the potential.

In addition, the result for the ψ' overlap integral (4) is sensitive to the charmonium wave function in the relativistic domain: only 60% of the integral comes from the momentum region $|\boldsymbol{p}| \leq 0.9m_c$ (the corresponding number for the J/ψ is 95%). This means that the theoretical calculation is not reliable for the ψ' amplitude, since the wave function was obtained from the non-relativistic Schrödinger equation. The sensitivity to relativistic momenta has previously been emphasized in Ref. [16].

We thus have to conclude that *there is no reliable theoretical prediction for the cross section ratio* $R_{el}^{\gamma N}$.

The weighted photon wave function $r_\perp^2 \Phi_\gamma$ probes $c\bar{c}$ pairs with $r_\perp \lesssim 0.5$ fm (*cf.* Fig. 1). Assuming that Eq. (4) is valid in this range implies that the light-cone charmonium wave functions we used are incorrect, since our result (Table I, first line) is a factor 2 to 5 below the data (2). On the other hand, the data may be used to constrain the physical wave functions. In particular,

- The range of the photon wave function narrows with Q^2, allowing a 'scan' of the wave functions [12].

- The nuclear target A-dependence of coherent J/ψ photoproduction is consistent with $\sigma^{hA} = A^\alpha \sigma^{hN}$, with $\alpha = 4/3$ [18]. This implies a small rescattering cross section, *ie.*, a small transverse size of the $c\bar{c}$ pairs which contribute to J/ψ production. It would be important to measure the A-dependence also in coherent ψ' production.

- In the case of *inelastic* J/ψ photoproduction only

one (Coulomb) gluon is exchanged with the target. The overlap integral corresponding to Eq. (4) then has a *single* power of $r_\perp$. This means that the charmonium wave function is effectively probed at lower values of $r_\perp$ than in elastic photoproduction. In our calculation using potential model wave functions the effect of changing the power of $r_\perp$ in the overlap integral is quite large, as shown in the last line of Table I.

- The mechanism of charmonium hadroproduction is still uncertain, but it is likely that a single gluon is exchanged with the target. (This is a feature of all present models [1,3,19]). Just as in inelastic photoproduction, one gluon exchange implies a single power of $r_\perp$ in the overlap integral (4). This is qualitatively consistent with the measured hadroproduction ratio (1), which is larger than $R_{el}^{\gamma N}$ but smaller than the ratio in large Q^2 electroproduction.

According to the above arguments, *we expect the inelastic photoproduction ratio to be the same as that measured in (inelastic) hadroproduction,* $\sigma(\psi')/\sigma_{dir}(J/\psi) \simeq$ .24. The preliminary data [14] on inelastic ψ' photoproduction is still too imprecise for a definite conclusion.

It is perhaps not so surprising that the potential model results for the charmonium wave functions are inaccurate. In particular, those models take no account of the fact that the ψ' lies only 44 MeV below $D\bar{D}$ threshold. It follows from the uncertainty principle that the ψ' wave function contains virtual $D\bar{D}$ pairs with a lifetime (and size) around 4 fm. The photoproduction amplitude, however, measures only the $|c\bar{c}\rangle$ component of the wave function. It is quite possible that this component is narrowly distributed in $r_\perp$, while $c\bar{c}$ pairs at larger separations are part of higher Fock states which contain gluons and light quarks.

ACKNOWLEDGMENTS

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